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A Recursive Least Square Approach to a Disturbance Observer Design for Balancing Control of a Single-wheel Robot System

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***Abstract*— Disturbance observer(DOB) has been known as a simple robust control method to reject disturbances. Design of Q filters is considered as the most important design technique for the DOB design to compensate for the possible improperness of the inverse model of the plant as well as a non minimum phase. Many efforts on Q-filter design have been accomplished to realize a stable filter for the inverse of the nominal model in the DOB. In this paper, a recursive least square approach to a disturbance observer (RbDOB) for modeling the plant is proposed. A recursive least square method is used to estimate model parameters of the 2nd order model through an input-output FIR filtering. Experimental studies of balancing control performances of a single-wheel robot by DOB and RbDOB are compared.**

***Index Terms*—Disturbance observer, model identification, one-wheel robot, recursive least square**

I. Introduction

Disturbances to the dynamical systems hinder them from behaving nicely as desired. Disturbance rejection is one of the main goals of robust control methods. Design of disturbance observer(DOB) is a simple and practical method to reject disturbances in the dynamical systems such as motion control systems. The robust performances of DOBs have been verified in many systems [1-8].

In the DOB design, the nominal model must be determined and its inverse model is used to extract the disturbance added control input to the system. Then the corrupted control input is compared with the known control input to identify the disturbance and it is finally cancelled by subtraction from the known control input.

During the DOB process, one of difficulties of DOB design is to satisfy the stability of the inverse model of the real plant. To guarantee the stability of the invers model, Q filter is designed and multiplied to compensate for the improperness of the inverse model as well as non minimum phase characteristics.

In general, Q filter is chosen as a special transfer function of a low pass filter type. Most of research on DOB-based control applications are focused on the Q filter design since how to design them are strongly related with the overall performances of DOB.

A variety of Q filter design has been presented in the literature. Among them three Q filter design methods are dominantly used [1,2]. One common point of them is to satisfy the stability for the inverse of the nominal model. For the Q filter design, three design factors such as a numerator order, a denominator order, and a time constant are concerned. The robustness of DOB is investigated and analyzed by considering the relation of design factors such as the order of a numerator, s denominator, and the order difference between them in the transfer function of Q filter [9].

To enhance the robustness, the manipulation of a nominal model has been presented [10]. The transfer function of the inverse of the nominal model is manipulated to extend the Q-filter bandwidth for more robustness. The tradeoff between the robustness and the sensitivity has to be considered, but it is not easy without considering a real plant. The performance of DOB is dependent upon the Q filter design which is strongly dependent upon the nominal plant model.

Therefore, in this paper, the identification of the plant is performed by the recursive least square (RLS) method along with DOB design. The RLS method has been effectively used in the parameter estimation of the real plant [11-16]. For simplicity, the plant model is assumed to be the 2nd order transfer function and the RLS identifies the corresponding filter coefficients.

The inverse of the nominal model is modified in the process to be realized considering the causality property of LTI(Linear Time Invariant) systems. However, a realizable form of the inverse through the RLS method can be derived. In that case, the stability and tuning problems are only remained in the DOB design. This provides a simplified solution compared to the traditional process of DOB design.

To achieve the stability of the proposed method, a novel and simple method is developed using a second order system example. A RbDOB(Recursive Least Square Based Disturbance Observer) is proposed as a novel concept for the effective concept for DOB based control systems. The coefficients of the second order system are estimated through a true input-output data of the real system. The data are used to identify the nominal model of the system by the RLS method. Based on the identified model, the performance of DOB is evaluated by experimental studies of balancing a single wheel robot system.

II. RBDOB

A. Review of RLS method

The RLS method is well known and widely used in the engineering application of system parameter identifications. The linear equation with real parameters $\boldsymbol{\theta}(t)$ can be described as

$$y(t) = \mathbf{x}^T \boldsymbol{\theta}(t) \quad (1)$$

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where

$$\mathbf{x}^T = [x(t), x(t-1), \cdots, x(t-n)] \quad (2)$$

$$\boldsymbol{\theta}^T = [b(0), b(1), \cdots, b(n)] \quad (3)$$

The equation can be described with the estimated parameters as

$$y(t) = \mathbf{x}^T\hat{\boldsymbol{\theta}}(t) + \hat{e}(t) \quad (4)$$

where, $\hat{\boldsymbol{\theta}}(t)$ are estimated parameters and $\hat{e}(t)$ is an error at time t.

$$\hat{e}(t) = y(t) - \mathbf{x}^T\hat{\boldsymbol{\theta}}(t)) \quad (5)$$

To form the least squares, $\hat{\boldsymbol{\theta}}(t)$ of (5) can be rewritten as

$$\hat{e}(t) = \mathbf{x}^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(t)) \quad (6)$$

The objective function to be minimized is given as

$$J = \sum_{t=1}^{N} \hat{e}^2 = \hat{\mathbf{e}}^T\hat{\mathbf{e}} \quad (7)$$

Optimal solution for the parameter can be obtained with gradient when the objective function becomes zero.

$$\frac{\partial}{\partial\hat{\boldsymbol{\theta}}} J = 0 \quad (8)$$

The solution of (8) can be obtained.

$$\hat{\boldsymbol{\theta}}(t) = [\mathbf{x}^T(t)\,\mathbf{x}(t)]^{-1}[\mathbf{x}^T(t)\,\mathbf{y}(t)] \quad (9)$$

Then, the following covariance matrix is defined.

$$\mathbf{P}(t) = [\mathbf{x}^T(t)\,\mathbf{x}(t)]^{-1} \quad (10)$$

Using a Matrix Inversion Lemma, we have the updated equations.

$$\mathbf{P}(t+1) = \mathbf{P}(t)(\mathbf{I} - \frac{x(t+1)x^T(t+1)\,\mathbf{P}(t)}{1 + x^T(t+1)\,\mathbf{P}(t)x(t+1)}) \quad (11)$$

$$\hat{\boldsymbol{\theta}}(t+1) = \hat{\boldsymbol{\theta}}(t) + \mathbf{P}(t+1)\,\mathbf{x}(t+1)(y(t+1) - \mathbf{x}^T(t+1)\,\hat{\boldsymbol{\theta}}(t)) \quad (12)$$

(11) and (12) with a forgetting factor λ can be rewritten.

$$\mathbf{P}(t+1) = \frac{1}{\lambda}\left(\mathbf{P}(t)(\mathbf{I} - \frac{x(t+1)x^T(t+1)\,\mathbf{P}(t)}{\lambda + x^T(t+1)\,\mathbf{P}(t)x(t+1)})\right) \quad (13)$$

where the memory index of the filter can be described as $\frac{1}{1-\lambda}$ and **I** is a unit matrix and it has the same number with the system parameters.

The overall process of the RLS scheme is given in Fig. 1.

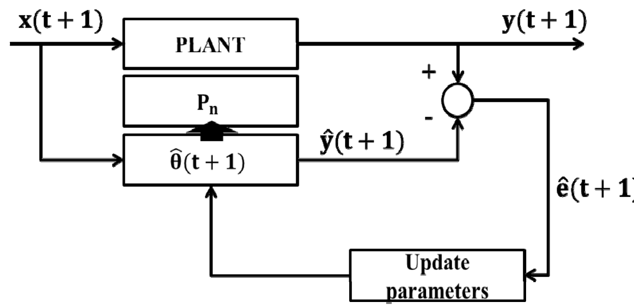

Fig. 1 RLS scheme.

Initial conditions are selected as

$$\boldsymbol{\theta}(0) = 0, \mathbf{P}(0) = \delta\,\mathbf{I} \quad (14)$$

where δ is set to a large value of 10000.

B. Model Identification and Stability

Here an unknown system is assumed as a second order system. Therefore, (1) can be described as follows.

$$y(n) = b_1x(0) + b_2x(1) + b_3x(3) \quad (15)$$

$$\boldsymbol{\theta}(n) = [b_1, b_2, b_3] \quad (16)$$

$$\mathbf{x}^T(n) = [x(0), x(1), x(2)] \quad (17)$$

The transfer function of the nominal model can be described as

$$P_n(z) = \frac{\hat{y}(z)}{x(z)} = b_1 + b_2z^{-1} + b_3z^{-2} \quad (18)$$

A FIR(Finite Impulse Response) filter is designed as a nominal model since the FIR filter is always stable. The inverse model is used as an input estimation filter where the dedicated Q-filter is coupled. Therefore, (19) is used in the DOB.

$$P_n^{-1}(z) = \frac{x(z)}{\hat{y}(z)} = \frac{1}{b_1 + b_2z^{-1} + b_3z^{-2}} \quad (19)$$

The selection of filter coefficients is done on the basis of the satisfaction of the stability.

C. Review of DOB

The conventional architecture of DOB can be described as in Fig. 2. In Fig. 2, r is the command input, u is the control input, d is the disturbance, y is the control output, ξ is the sensor noise, and $\hat{d}$ is the estimated disturbance. Traditionally, DOB is aimed to cancel the disturbance d by estimating $\hat{d}$ through filtering the output based on the nominal model of the plant.

From Fig. 2, the output is described as

$$y = \frac{PP_n}{P_n + (P - P_n)Q} v + \frac{PP_n(1-Q)}{P_n + (P - P_n)Q} d + \frac{PQ}{P_n + (P - P_n)Q} \xi \qquad (20)$$

Q filter is selected as a lowpass filter such that $Q(s) \approx 1$ to cancel the disturbance effect at low frequencies and $Q(s) \approx 0$ to eliminate the sensor noise effect at high frequencies. To realize the nominal model under the stable property, Q-filter must be designed with care.

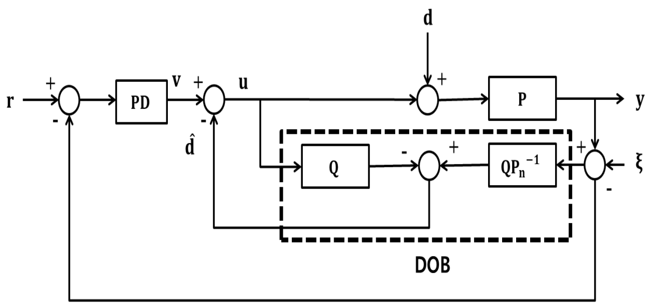

Fig. 2 DOB structure

D. RLS-based DOB (RbDOB)

There are three major methods for designing Q filters in the literature [1,2,9]. The common goal of Q filter design is to satisfy the stability, disturbance rejection and the sensor noise immunity. When the bandwidth of Q is enlarged, the disturbance rejection property is increased. However, the noise immunity property of it is diminished due to the complementary characterictics of the filter.

Therefore, here Q filter design can be omitted by estimating the nominal model of the plant through the RLS method. We assume that the RLS approximation of the plant can closely describe the plant.

Based on the concept, a RLS-based DOB is proposed and its overall scheme is shown in Fig. 3. Instead of using a Q filter, the forgetting factor of the RLS method is used as a parameter that can adjust the cross point of Q and (1-Q).

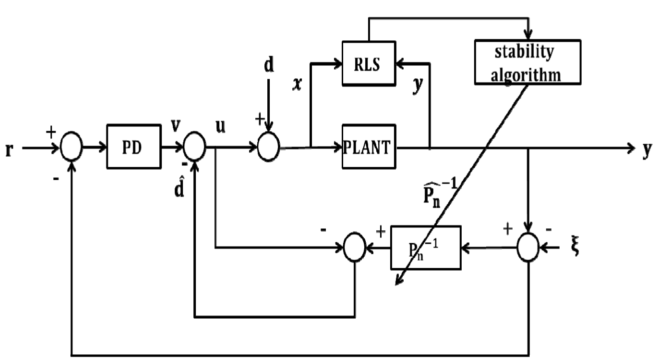

Fig. 3 Proposed RbDOB structure.

III. Experimental Study

A. Experimental setup

The feasibility of the proposed RbDOB is tested for balancing control of a one-wheel mobile robot system. The robot can maintain balance using a gyroscopic torque induced from the gimbal system. The output of the one-wheel robot system is the balancing angle, which is a roll angle when the torque input is given. Therefore, the system is assumed as a simple SISO(Single Input Single Output) system to verify the proposed control concept.

In Fig. 4, x is the control input such as a torque and y is the control output such as a roll angle of the robot system. The real system of the robot is shown in Fig. 4 [18-19].

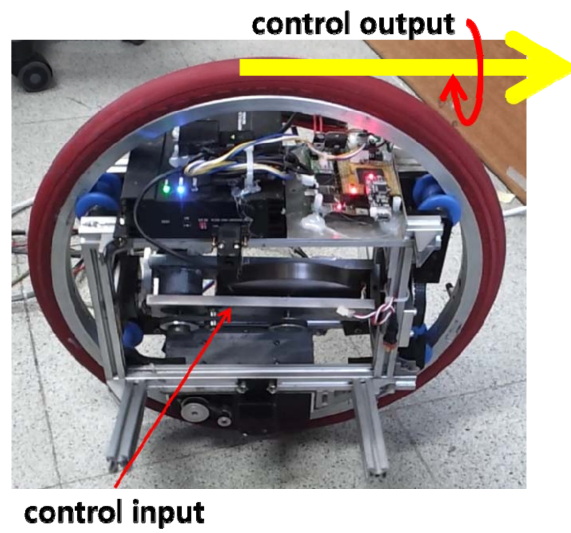

Fig. 4 One-wheel robot system.

The single-wheel robot is aimed to maintain balance and drive itself on its terrain. The main actuator is the gyroscopic force induced by the gimbal system that is composed of a flywheel, a rotating motor, and a tilt motor. Gyroscopic motion is controlled by the rate of tilting the gimbal system.

Therefore, the input and the output relationship can be approximately modeled as SISO although the robot is a complicated system. The input and the output for the robot are simply the tilting and the roll angle, respectively. The gyroscopic actuator is the control input and the roll motion is the control output.

B. Experimental results

1) Identification

To verify the feasibility of RbDOB, both input and output data in the real control situation are obtained. The saturation value of the input x was about $\pm 7(\mathrm{V})$. Under the conditions, we log the output signal at the same time. The number of collected data are 1,094 every 100ms. From the input/output data, the coefficients are estimated by using the RLS algorithm. The forgetting factor of 0.995 is used for the experimental studies.

Fig. 5 shows the real and the estimated performances which turn out to be very comparable.

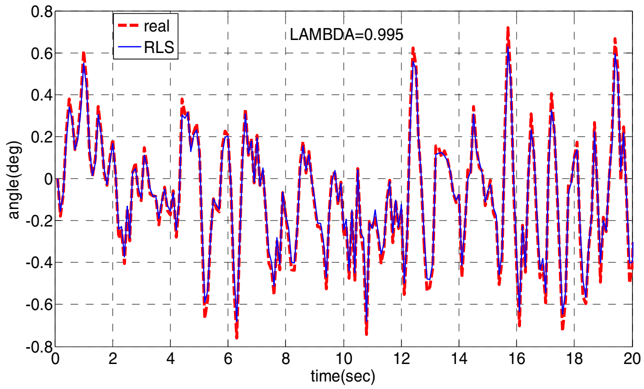

Fig. 5 Proposed RLS performance.

The estimated coefficients from the RLS method to generate the estimated output are found such that FIR filter becomes

$$P_n(z) = \frac{\hat{y}(z)}{x(z)} = 0.0543 + 0.0436z^{-1} + 0.027z^{-2} \tag{21}$$

2) Performance between DOB and RbDOB

Next experiment is to compare the balancing control performances between DOB and RbDOB. Fig. 6 shows the balancing angles of two control schemes. We see that both schemes maintain balance well. However, the balancing error of RbDOB is much less than that of DOB.

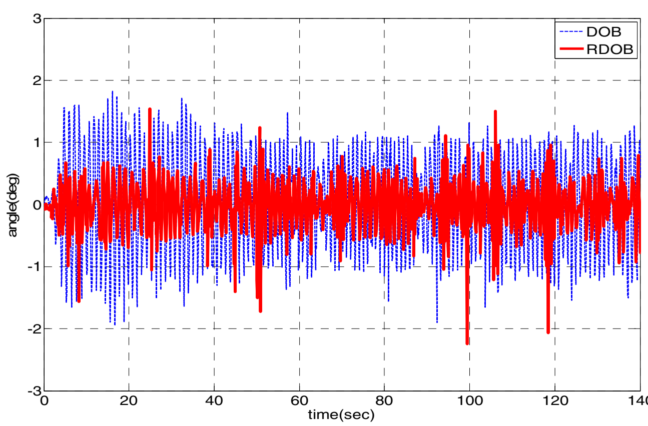

Fig. 6 Performance comparison between DOB and RbDOB

IV. Conclusion

Instead of designing a Q filter, the recursive least square method was used to identify the system model parameters. We found that the forgetting factor might be used as a tuning parameter for the performance like the function of a time constant in Q filters. The second order FIR filter identified by the RLS is aimed to guarantee the stable property of the inverse model required in the DOB design. The balancing control performance of a single-wheel robot by the proposed RbDOB has turned out to be better than that of DOB empirically. More investigation on the effect of the forgetting factor and the filter order will be explored in the future.

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References

[1] K. Ohnishi, M. Shibata, and T. Murakami, “Motion control for advanced mechatronics. Mechatronics”, *IEEE/ASME Transactions on Mechatronics*, 1(1), 1996, 56-67.

[2] T. Umeno, Y. Hori, “Robust speed control of DC servomotors using modern two degrees-of-freedom controller design”, *IEEE Transactions on Industrial Electronics*, 38(5), 1991, 363-368.

[3] T. Umeno, Y. Hori, “Robust servosystem design with two degree of freedom and its application to novel motion control of robot manipulators”, *IEEE Transactions on Industrial Electronics*, 40(5), 1993, 473-485.

[4] A. Tesfaye, H. S. Lee, and M. Tomizuka, “A sensitivity optimization approach to design of a disturbance observer in digital motion control systems”, *IEEE/ASME Transactions on Mechatronics*, 5(1), 2000, 32-38.

[5] M. Ruderman, “Tracking control of motor drives using feedforward friction observer”, *IEEE Transactions on Industrial Electronics*, 61(7), 2014, 3727-3735.

[6] E. Sariyildiz, K. Ohnishi, “On the explicit robust force control via disturbance observer. Industrial Electronics”, *IEEE Transactions on Industrial Electronics*, 62(3), 2015, 1581-1589.

[7] W. T. Su, C. M. Liaw, “Adaptive positioning control for a LPMSM drive based on adapted inverse model and robust disturbance observer”, *IEEE Transactions on Power Electronics*, 21(2), 2006, 505-517.

[8] W. H. Chen, J. Yang, L. Guo, and S. Li, “Disturbance-observer-based control and related methods-an overview”, *IEEE Transactions on Industrial Electronics*, 63(2), 2016, 1083-1095.

[9] Y. Choi, K. Yang, W. K. Chung, H. R. Kim, I. H. Suh, “On the robustness and performance of disturbance observers for second-order systems”, *IEEE Transactions on Automatic Control*, 48(2), 2003, 315-320.

[10] K. Kong, M. Tomizuka, “Nominal model manipulation for enhancement of stability robustness for disturbance observer-based systems”, *International Journal of Control, Automation, and Systems*, 11(1), 2013, 12-20.

[11] H. Unbehauen, G. P. Rao, “A review of identification in continuous-time systems”, Annual Reviews in Control, 22, 1998, 145-171.

[12] B. Bamieh, L. Giarre, “Identification of linear parameter varying models”, *International Journal of Robust and Nonlinear Control*, 12(9), 2002, 841-853.

[13] Y. Hu, “Iterative and recursive least squares estimation algorithms for moving average systems”, *Simulation Modelling Practice and Theory*, 34, 2013, 12-19.

[14] C. Wang, T. Tang, “Recursive least squares estimation algorithm applied to a class of linear-in-parameters output error moving average systems”, *Applied Mathematics Letters*, 29, 2014, 36-41.

[15] R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne, “Design issues in adaptive control”, *IEEE Transactions on Automatic Control*, 33(1), 1988, 50-58.

[16] C. E. Davila, “An efficient recursive total least squares algorithm for FIR adaptive filtering”, *IEEE Transactions on Signal Processing*, 42(2), 1994, 268-280.

[17] S. D. Lee, S. Jung, “Analysis of relationship between body and gimbal motion through experiment of a single-wheel robot based on an inverse gyroscopic effect”, *International Journal of Control, Automation, and Systems*, 21(11), 2015, 1064-1069.

[18] S. D. Lee, S. Jung, “Experimental verification of stability region of balancing a single-wheel robot: An inverted stick model approach”, *IECON* 2015, 2015, 4556-4561.